

# Josephson Effect in Magnetic Superconductors with Spiral Magnetic Order

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It is shown that in magnetic superconductors with spiral magnetic order the Josephson current has an additional contribution which depends: (i) on the relative orientation (magnetic phase)  $\theta = \theta_L - \theta_R$  of magnetizations on the left ( $L$ ) and right ( $R$ ) banks of the contact, (ii) on the junction helicity  $\chi = \chi_L \chi_R$ , (with spiral helicity  $\chi_{L(R)} = \pm 1$ ), i.e.  $J = [J_c - J_\chi \cos \theta] \sin \varphi$  with  $\varphi = \varphi_L - \varphi_R$ . The ratio  $R_\chi \equiv J_\chi / J_c$  is calculated as a function of the superconducting order parameter  $\Delta$ , the exchange field energy  $h$  and the wave vector  $Q$  of the spiral magnetic structure. The  $\pi$ -Josephson contact can be realized in such a system in some region of parameters. Some possible consequences of this new phase relation is also analyzed.

*Introduction* - The physics of magnetic superconductors (MSC) is interesting due to competition of magnetic order and singlet superconductivity (SC) in bulk materials. The problem of their coexistence was first studied theoretically by V. L. Ginzburg [1] in 1956, while the experimental progress begun after the discovery of ternary rare earth (RE) compounds (RE)Rh<sub>4</sub>B<sub>4</sub> and (RE)Mo<sub>6</sub>X<sub>8</sub> (X=S,Se) [2] with regular distribution of localized RE magnetic ions. It turned out that in many of these systems SC, with the critical temperature  $T_c$ , coexists rather easily with antiferromagnetic (AF) order, with the critical temperature  $T_{AF}$ , where usually one has  $T_{AF} < T_c$  [2]. Due to their antagonistic characters singlet SC and ferromagnetic (FM) order can not coexist in bulk samples with realistic physical parameters, but under certain conditions the FM order, in the presence of SC, is transformed into a spiral or domain-like structure - depending on the anisotropy of the system [4], [3]. These two orderings coexist in a limited temperature interval  $T_{c2} < T < T_m$  (reentrant behavior in ErRh<sub>4</sub>B<sub>4</sub> and HoMo<sub>6</sub>S<sub>8</sub>) or even down to  $T = 0$  K (in HoMo<sub>6</sub>Se<sub>8</sub>). Note, in ErRh<sub>4</sub>B<sub>4</sub> one has  $T_c = 8.7$  K,  $T_m \approx 0.8$  K,  $T_{c2} \approx 0.7$  K, while for HoMo<sub>6</sub>S<sub>8</sub> one has  $T_c = 1.8$  K,  $T_m \approx 0.74$  K,  $T_{c2} \approx 0.7$  K - see Refs. [2], [3], [4]. In most new quaternary compounds (RE)Ni<sub>2</sub>B<sub>2</sub>C the AF order and SC coexist [5], while in HoNi<sub>2</sub>B<sub>2</sub>C, an additional oscillatory magnetic order exists competing strongly with SC leading to reentrant behavior [6]. Recently, the coexistence of SC and nuclear magnetic order was observed in AuIn<sub>2</sub> [7], where  $T_c = 0,207$  K and  $T_m = 35$   $\mu$ K, which was explained in terms of spiral or domain-like magnetic structure [8]. There are evidences for the coexistence of the AF (or FM) order and SC ( $T_{AF} = 137$  K,  $T_c < 45$  K) in layered perovskite superconductor RuSr<sub>2</sub>GdCu<sub>2</sub>O<sub>8</sub> [9], where AF and SC orders are spatially separated in Ru-O and Cu-O planes, respectively.

In all of the above MSC systems the exchange(EX) interaction between localized magnetic moments (LM's) and SC is much larger (the EX model) than the electromagnetic (EM) one. The latter is due to the orbital effect of the magnetic induction  $\mathbf{B} \approx 4\pi\mathbf{M}$  on SC electrons.

In the following we study the Josephson effect in MSC with spiral magnetic order in the framework of the EX model. It is shown below that the Josephson current depends additionally on the magnetic phase  $\theta = \theta_L - \theta_R$  of the magnetic order parameters, as well as on the spiral helicities  $\chi_{L(R)} = \pm 1$  on the left and right surface.

The microscopic theory of MSC [3] takes into account the interaction between LM's and conduction electrons which goes via: *a*) the direct EX interaction and *b*) the EM interaction where the dipolar magnetic field  $\mathbf{B}_m(\mathbf{r}) = 4\pi\mathbf{M}(\mathbf{r})$  created by LM's acts on the orbital motion of electrons. The Hamiltonian of the system has the form

$$\begin{aligned} \hat{H} = \hat{H}_0 + \hat{H}_{BCS} + \int d^3r \{ \hat{\psi}^\dagger(\mathbf{r}) \hat{V}_{ex}(\mathbf{r}) \hat{\psi}(\mathbf{r}) + \\ + \frac{[\text{curl}\mathbf{A}(\mathbf{r})]^2}{8\pi} \} + \sum_i [-\mathbf{B}(\mathbf{r}_i) g\mu_B \hat{\mathbf{J}}_i]. \end{aligned} \quad (1)$$

$\hat{H}_0 \equiv \hat{H}_0(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A})$  describes the motion of quasiparticles in the magnetic field  $\mathbf{B}(\mathbf{r}) = \text{curl}\mathbf{A}(\mathbf{r})$ , which is due to LM's and screening superconducting current.  $\hat{H}_{BCS} \equiv \hat{H}_{BCS}(\Delta(\mathbf{r}))$  is the BCS pairing Hamiltonian with the SC order parameter  $\Delta(\mathbf{r})$ ,  $\hat{V}_{ex}(r) =$

$\hat{\mathbf{h}}(\mathbf{r})\sigma$  is the EX potential where  $\hat{\mathbf{h}}(\mathbf{r}) = \sum_i J_{ex}(\mathbf{r} - \mathbf{r}_i)(g - 1)\hat{\mathbf{J}}_i$ . Here,  $J_{ex}(\mathbf{r})$  is the exchange integral between electronic spins  $\sigma$  and LM's and  $\hat{\mathbf{J}}_i$  is the total angular momentum operator of the LM at the  $i$ -th site and  $\sigma = (\sigma_1, \sigma_2, \sigma_3)$  are Pauli spin matrices. In case of magnetic anisotropy the crystal-field term  $\hat{H}_{CF}(\hat{\mathbf{J}}_i)$  should be added to  $\hat{H}$ .

In the following a clean  $s$ -wave MSC is considered with spiral magnetic order with the wave vector  $\mathbf{Q}$  along the  $z$ -axis,  $\mathbf{Q} = Q_z\hat{\mathbf{z}} = \pm Q\hat{\mathbf{z}}$ , and the spiral helicity  $\chi = Q_z/Q = \pm 1$ . The LM's are assumed to lie in the  $xy$ -plane due to the easy-plane magnetic anisotropy and the mean-field EX potential  $\hat{V}(\mathbf{r}) \equiv \langle \hat{V}_{ex}(\mathbf{r}) \rangle$  reads

$$\hat{V}(\mathbf{r}) = \begin{pmatrix} 0 & he^{-i(\chi Qz + \theta)} \\ he^{i(\chi Qz + \theta)} & 0 \end{pmatrix}. \quad (2)$$

Here,  $h = n_m(g - 1)J_{ex}(0) \langle |\hat{\mathbf{J}}| \rangle$  and  $J_{ex}(0)$  is the  $q = 0$  Fourier component of  $J_{ex}(\mathbf{r} - \mathbf{r}_i)$ ,  $n_m$  is the concentration of regularly distributed LM's - see [3]. For further purposes we define  $h_\theta = he^{i\theta}$  where the magnetic phase  $\theta$  characterizes the orientation of the EX field at the surface  $z = 0$ . The Josephson current between two MSC, both with spiral magnetic order, depends on anomalous Green's functions  $F_{\sigma_1\sigma_2}^\dagger(x_1, x_2) = \langle \hat{T}\hat{\psi}_{\sigma_1}^\dagger(x_1)\hat{\psi}_{\sigma_2}(x_2) \rangle$  with  $x = (\mathbf{r}, \tau)$  and  $\sigma = \uparrow, \downarrow$ , which are calculated in [4]

$$F_{\uparrow\uparrow}^\dagger(\mathbf{k}, \mathbf{k}'; \omega_n) = \delta(\mathbf{k} - \mathbf{k}')\Delta \frac{\omega_n^2 + \Delta^2 - h^2 + (\varepsilon_{\mathbf{k}} + \rho_{\mathbf{k}})^2}{[\omega_n^2 + E_{1\mathbf{k}}^2][\omega_n^2 + E_{2\mathbf{k}}^2]}. \quad (3)$$

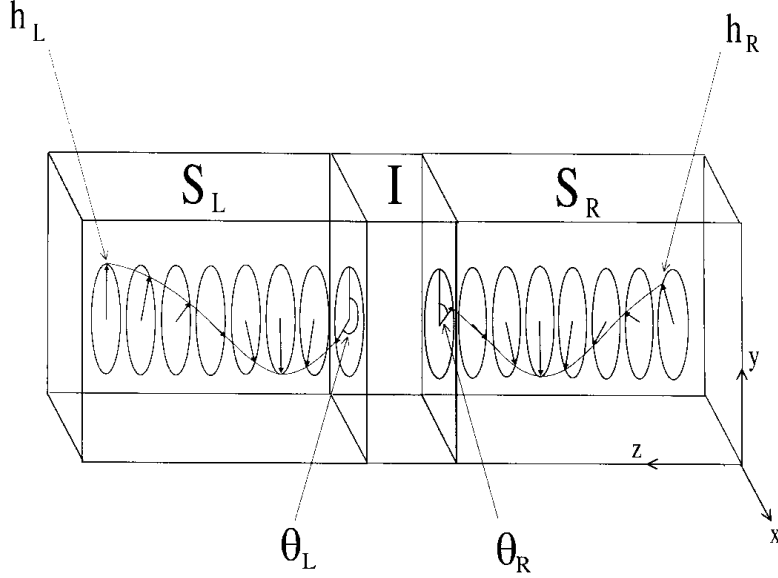
$$F_{\uparrow\uparrow}^\dagger(\mathbf{k} + \mathbf{Q}, \mathbf{k}'; \omega_n) = -\delta(\mathbf{k} - \mathbf{k}')2\Delta h_\theta^* \frac{[i\omega_n - \rho_{\mathbf{k}}]}{[\omega_n^2 + E_{1\mathbf{k}}^2][\omega_n^2 + E_{2\mathbf{k}}^2]}. \quad (4)$$

The excitation spectrum  $E_{1,2\mathbf{k}}$  is given by  $E_{1,2\mathbf{k}}^2 = \varepsilon_{\mathbf{k}}^2 + \rho_{\mathbf{k}}^2 + h^2 \pm |\Delta|^2 \mp 2\sqrt{\varepsilon_{\mathbf{k}}^2(\rho_{\mathbf{k}}^2 + h^2) + |\Delta|^2 h^2}$ , where  $\varepsilon_{\mathbf{k}} = [\xi(\mathbf{k} + \mathbf{Q}/2) + \xi(\mathbf{k} - \mathbf{Q}/2)]/2$  and  $\rho_{\mathbf{k}} = [\xi(\mathbf{k} + \mathbf{Q}/2) - \xi(\mathbf{k} - \mathbf{Q}/2)]/2$ , i.e.  $\varepsilon_{\mathbf{k}} = \xi(\mathbf{k})$  and  $\rho_{\mathbf{k}} = \mathbf{v}_F \cdot \mathbf{Q}$ . The pair function  $F_{\downarrow\downarrow}^\dagger$  is obtained from  $F_{\uparrow\uparrow}^\dagger$  by replacing  $h_\theta^* \rightarrow h_\theta$ , i.e.  $\theta \rightarrow -\theta$  and  $\mathbf{Q} \rightarrow -\mathbf{Q}$ . In the following we assume that  $Q \ll k_F$ . Note that in the MSC system with spiral magnetic order triplet pairs with amplitudes  $F_{\uparrow\uparrow}^\dagger$  and  $F_{\downarrow\downarrow}^\dagger$  (which depend on  $h_\theta$ ) live short time inspite of the absence of triplet pairing. Below is demonstrated that the finite life-time (of the order  $\hbar/\Delta$ ) of the triplet pairs leads to nontrivial effects in the Josephson current.

The above theory [3], [4] predicts that if the EX interaction dominates then SC modify the FM order into the spiral with the wave vector  $Q$ ,  $\xi_0^{-1} \ll Q \ll k_F$ , and with zero average EX field  $\langle \mathbf{h} \rangle_{\xi_0} \approx 0$  ( $\xi_0$  is the coherence length). The two ordering coexist in a narrow temperature interval in  $\text{ErRh}_4\text{B}_4$ ,  $\text{HoMo}_6\text{S}_8$ , or up to  $T = 0$  K in  $\text{HoMo}_6\text{Se}_8$ . In  $\text{AuIn}_2$  SC coexists with the nuclear (spiral) magnetic order up to  $T = 0$  K [7], [8].

*Josephson current* - We consider tunneling contact (MSC-I-MSC) between two (left- $L$  and right- $R$ ) MSC both with spiral magnetic ordering with the wave vector  $\mathbf{Q}_{L,R}$ , the SC order parameter  $\Delta_{L,R} = |\Delta_{L,R}| e^{i\varphi_{L,R}}$  and the EX field  $h_{\theta_{L(R)}} = h_{L(R)} e^{i\theta_{L(R)}}$ , respectively - see Fig.1. For simplicity it is assumed that: (i)  $|\Delta_L| = |\Delta_R| = |\Delta|$ ,  $h_L = h_R = h$  while  $\varphi = \varphi_L - \varphi_R \neq 0$  and  $\theta = \theta_L - \theta_R \neq 0$ ; (ii)  $|\mathbf{Q}_L| = |\mathbf{Q}_R| = Q$  where  $\mathbf{Q}_{L,R} = \chi_{L,R}Q\hat{\mathbf{z}}$  are orthogonal to the tunneling barrier, where helicities of MSC are  $\chi_{L(R)} = \pm 1$  for  $\mathbf{Q}_{L,R}$  along (+) or opposite(-) to the  $z$ -axis. In that case one has

$\rho_{\mathbf{k},\mathbf{L}(\mathbf{R})} = \mathbf{v}_F \cdot \mathbf{Q}_{L(R)} = \chi_{L(R)}\rho = \chi_{L(R)}\rho_0 \cos \beta$ , where  $\beta$  is the angle between  $\mathbf{v}_F$  and  $z$ -axis.



**FIGURE 1.** The Josephson contact  $I$  between two magnetic superconductors  $S_L$  and  $S_R$  with spiral magnetic orders. The corresponding exchange fields  $h_{L,R}$  at the surface make angles  $\theta_{L,R}$  with the  $y$ -axis. The wave vectors  $\mathbf{Q}_{L,R}$  of the spirals are along the  $z$ -axis

The tunneling of a left-side electron with momentum and spin  $(\mathbf{k}_L, \sigma)$  into a right-side one  $(\mathbf{k}_R, \sigma)$  is described by the standard tunneling Hamiltonian [10]. For simplicity it is assumed, as usual, that the tunneling amplitude  $T_{\mathbf{k}_L, \mathbf{k}_R}$  is weakly energy and momentum dependent, i.e.  $T_{\mathbf{k}_L, \mathbf{k}_R} \approx T_0 \Theta(k_{L,z} k_{R,z})$ . The Heaviside function  $\Theta$  takes into account that before tunneling the left electron moves toward the barrier, while after it moves as the right electron away from the barrier, and vice versa. The assumed  $T_{\mathbf{k}_L, \mathbf{k}_R}$  is more suitable for diffusive barrier (incoherent tunneling). The extension of the theory to the momentum dependence  $T_{\mathbf{k}_L, \mathbf{k}_R}$  is straightforward.

The standard theory of the Josephson effect [10] applied to MSC systems gives the Josephson current  $J(\varphi, \theta)$  and the energy of the junction  $E_J(\varphi, \theta)$

$$J(\varphi, \theta) = [J_c - J_\chi \cos \theta] \sin \varphi, \quad (5)$$

$$E_J(\varphi, \theta) = -\frac{\Phi_0 J_c}{2\pi c} [1 - R_\chi \cos \theta] \cos \varphi + \text{const}, \quad (6)$$

where,  $\varphi = \varphi_L - \varphi_R$ ,  $\theta = \theta_L - \theta_R$ ,  $\Phi_0$  is the flux quantum and  $R_\chi = J_c/J_\chi$ . The first term in the bracket ( $\sim J_c$ ) is the standard one due to the singlet pair tunneling  $\sum_{\mathbf{k}_L, \mathbf{k}_R, \omega_n} |T_{\mathbf{k}_L, \mathbf{k}_R}|^2 F_{\uparrow\downarrow, L}^\dagger(\mathbf{k}_L, \omega_n) F_{\uparrow\downarrow, R}(\mathbf{k}_R, -\omega_n)$ . In the calculation the summation over  $\mathbf{k}$  is replaced by the integration over  $\xi$  and  $\rho (\equiv \rho_0 \cos \beta = \rho_0 y)$ , i.e.

$$\sum_{\mathbf{k}} (...) = \frac{N(0)}{2\rho_0} \int^{\rho_0} d\rho \int_{-\infty}^{\infty} d\xi (...) \quad (7)$$

$N(0)$  is the density of states on the Fermi level. After the integration over  $\xi$  one obtains  $J_c$  (in the following  $|\Delta| \equiv \Delta$ )

$$J_c = 4e\pi^2 N^2(0) |T_0|^2 \Delta^2 T \sum_{n=1}^{\infty} \left[ \int_0^1 I(\omega_n, y) dy \right]^2$$

$$I(\omega_n, y) = \frac{a_n + \sqrt{a_n^2 - 4\Delta^2 h^2} - 2h^2}{\sqrt{a_n^2 - 4\Delta^2 h^2} \sqrt{a_n - 2\rho_0^2 y^2 - 2h^2 + \sqrt{a_n^2 - 4\Delta^2 h^2}}}. \quad (8)$$

$a_n = \omega_n^2 + \rho_0^2 y^2 + h^2 + \Delta^2$ ,  $\rho_0 = Qv_F$ ,  $v_F$  is the Fermi velocity and  $T$  is the temperature.

The second term in Eq.(5) ( $\sim J_\chi$ ) depends on the relative magnetic phase  $\theta = \theta_L - \theta_R$  of the EX fields at the barrier surfaces, and on the Junction helicity  $\chi (\equiv \chi_L \chi_R) = \pm 1$ . Both are due to the tunneling of Cooper pairs being short time in the triplet state.  $J_\chi$  is due to the term  $\sum_{\mathbf{k}_L, \mathbf{k}_R, \omega_n} |T_{\mathbf{k}_L, \mathbf{k}_R}|^2 [F_{\uparrow\uparrow, L}^\dagger(\mathbf{k}_L, \omega_n) [F_{\uparrow\uparrow, R}^\dagger(\mathbf{k}_R, -\omega_n)]^* + F_{\downarrow\downarrow, L}^\dagger(\mathbf{k}_L, \omega_n) [F_{\downarrow\downarrow, R}^\dagger(\mathbf{k}_R, -\omega_n)]^*]$ . After the  $\xi$ -integration  $J_\chi$  reads

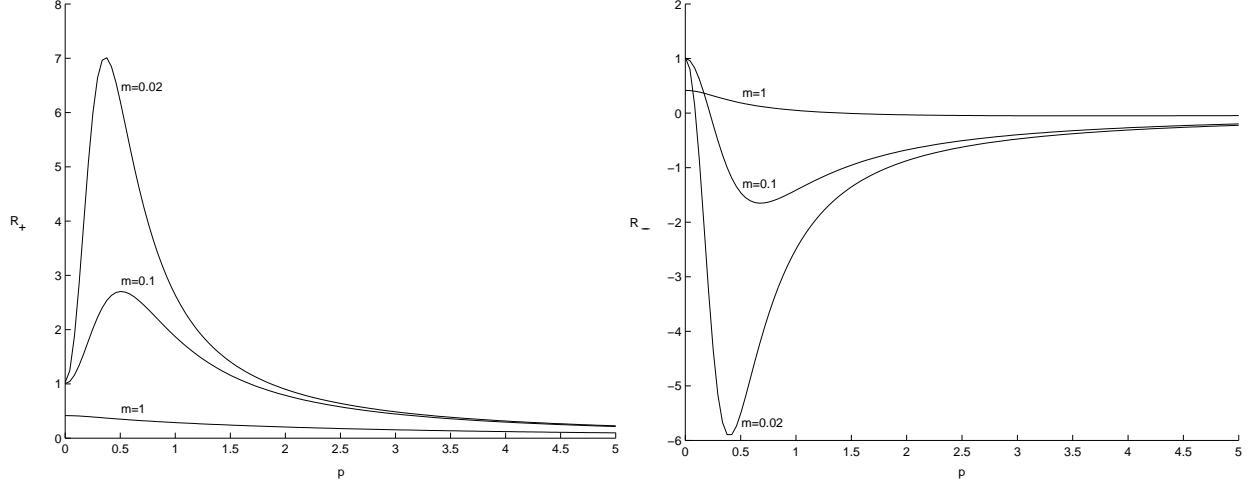
$$J_\chi = 16e\pi^2 N^2(0) t_0^2 \Delta^2 h^2 T \sum_{n=0}^{\infty} \left\{ [\omega_n \int_0^1 K(\omega_n, y) dy]^2 + \right.$$

$$\left. + \chi_L \chi_R [\rho_0 \int_0^1 y K(\omega_n, y) dy]^2 \right\},$$

$$K(\omega_n, y) = \frac{I(\omega_n, y)}{a_n + \sqrt{a_n^2 - 4\Delta^2 h^2} - 2h^2}. \quad (9)$$

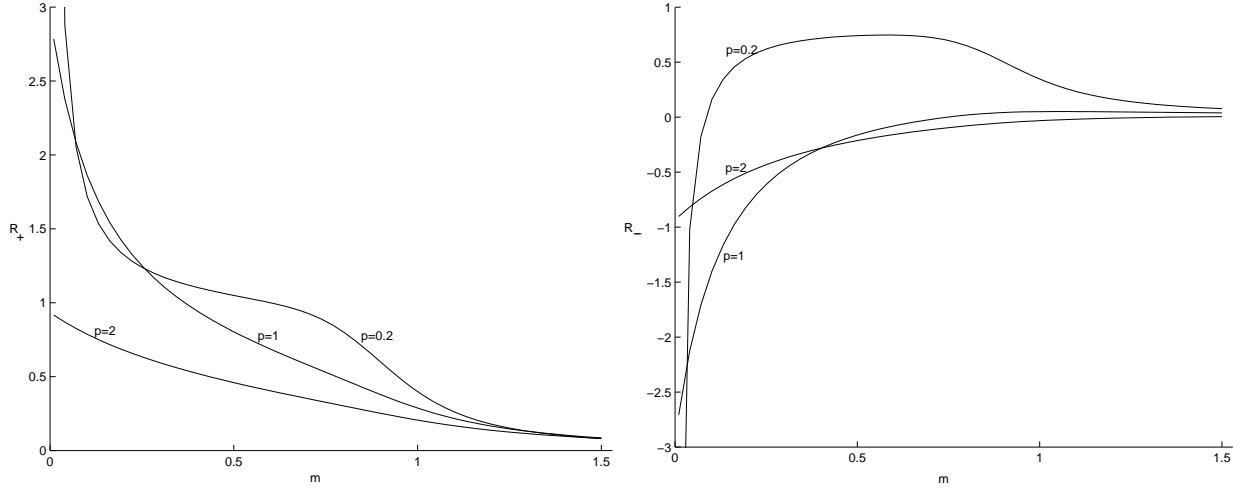
From Eq.(9) follows that  $J_\chi = 0$  for  $h = 0$ , while the standard Josephson current  $J_c$  is finite at  $h = 0$  reaching its maximum.  $J_\chi > 0$  for the total Junction helicity  $\chi (\equiv \chi_L \chi_R) = 1$ , while for  $\chi = -1$  it can be negative depending on  $\Delta, h, \rho_0$  - see discussion and Figs.2b, 3b below. In order to calculate  $J_c$  and  $J_\chi$  equilibrium values of  $\Delta, h, Q$ , which minimize the free-energy  $F(\Delta, h, Q)$ , are needed. As it is shown in [3]  $F(\Delta, h, Q)$  (and  $\Delta, h, Q$ ) depends on microscopic parameters  $k_F, v_F, n_m, J_{ex}, \Delta_0$ , lattice structure, etc. what shall be not studied here.

In the following the ratio  $R_\chi(m, p) = J_\chi/J_c$  is calculated numerically as a function of  $m = \Delta/h$  and  $p = (\rho_0/h) (= Qv_F/h)$  at the temperature  $T = 0.1\Delta$ .  $m$  and  $p$  are supposed to be equilibrium values. The results are shown in Fig.2 for  $p = \text{const}$



**FIGURE 2.** The ratio  $R_\chi(m = \text{const}, p) = J_\chi/J_c$  for various  $m = 1; 0.1; 0.02$ . (a)  $\chi = +1$ ; (b)  $\chi = -1$ . For  $|R_\chi| > 1$  the  $\pi$ -contact is realized.

and in *Fig.3* for  $m = \text{const}$ .



**FIGURE 3.** The ratio  $R_\chi(m, p = \text{const}) = J_\chi/J_c$  for various  $p = 2; 1; 0.2$ . (a)  $\chi = +1$ ; (b)  $\chi = -1$ . For  $|R_\chi| > 1$  the  $\pi$ -contact is realized.

### Discussion

(i)  $\pi$ -junction: - The above theory predicts that, besides  $|R_\chi| < 1$ , the case  $|R_\chi| > 1$  can be realized depending on  $m$  and  $p$  - see *Figs.(2 – 3)*. From *Eqs.(5,6)* comes out that for  $|R_\chi| > 1$  the  $\pi$ -junction is realized, i.e.  $\min E_J$  is reached for  $\varphi = \pi$ . *Eq.(5,6)* imply also that for  $\chi = -1$  the case  $R_\chi < -1$  is realized in some region of  $m$  and  $p$  (see *Figs.(2-3)*) and the  $\pi$ -contact is realized for those  $\theta$  with  $\cos \theta < -1/|R_\chi|$ . For  $R_\chi > 1$  the  $\pi$ -contact is realized for  $\cos \theta > 1/|R_\chi|$ . The favorable range of parameters  $m$  and  $p$  for the realization of  $\pi$ -junction are seen from *Figs.(2 – 3)* and it lies in the region  $m < 1$ ,  $p < 2$ . Note that  $m(\equiv \Delta/h_{ex})$  is a measure of the strength of the SC order parameter with respect to the magnetic order parameter (exchange energy), while  $p(\equiv Qv_F/h_{ex}) = l_{ex}/L_{spiral}$  measures the inverse of spiral period ( $L_{spiral} = 2\pi/Q$ ). This means that MSC systems with spiral magnetic order which are more ferromagnetic-like

are favorable for the realization of the  $\pi$ -contact. On the other side the latter property is less favorable for the coexistence of SC and spiral magnetic order. In that respect it is worth of mentioning that in ferromagnetic SC  $\text{ErRh}_4\text{B}_4$ ,  $\text{HoMo}_6\text{S}_8$ ,  $\text{HoMo}_6\text{Se}_8$  and  $\text{AuIn}_2$ ,  $m$  varies from  $m > 1$  to  $m \ll 1$  by lowering  $T$  from  $T_m$ , while  $p \gg 10$  is realized thus making  $|R_\chi|$  small in these systems [3].

The proposed theory holds also for MSC with AF magnetic order (the limiting case of the spiral order) with an easy-plane magnetic anisotropy. For instance in some AF heavy-fermion superconductors, like  $\text{URu}_2\text{Si}_2$  with  $T_{AF} \approx 17 \text{ K}$  and  $T_c \approx 1.5 \text{ K}$ , one has  $p \approx 1 - 2$ ,  $m \approx (0.01 - 0.03)$ , where small value of  $p$  is due to the small Fermi velocity [11]. If one assumes that in this system (anisotropic) s-wave SC is realized in absence of the AF order, then the latter changes SC in the way described above giving rise to short living triplet pairs, gapless superconductivity, power low behavior, etc. [12]. Since in  $\text{URu}_2\text{Si}_2$  one has  $p \approx 1 - 2$ ,  $m \approx (0.01 - 0.03)$  and  $|R_\chi| \gtrsim 1$  this means that it is favorable for making  $\pi$ -junctions. In that respect other heavy fermions, like  $\text{UPd}_2\text{Al}_3$  ( $T_{AF} \approx 14 \text{ K}$ ,  $T_c \approx 2 \text{ K}$ ) and  $\text{UNi}_2\text{Al}_3$  ( $T_{AF} \approx 4.6 \text{ K}$ ,  $T_c \approx 1 \text{ K}$ ), might belong to the class described by the above theory [12].  $\text{UPt}_3$  ( $T_{AF} \approx 5 \text{ K}$ ,  $T_c \approx 0.5 \text{ K}$ ) is also a candidate for such a  $\pi$ -junction, if it can be described by the above theory, because  $p \approx 1 - 2$ ,  $m \approx (0.01 - 0.03)$  in it. However, various experiments in  $\text{UPt}_3$  are well described by the unconventional superconductivity [11] - the type of order parameter is still under debate.

(ii) *magnetic phase  $\theta$* : In the case of an easy-plane (x-y plane) magnetic anisotropy the magnetic phase difference  $\theta (\equiv \theta_L - \theta_R)$  can be tuned in the range  $(\pi, -\pi)$ , for instance, by applying magnetic field on lateral surfaces of the left and right superconducting banks. So if the parameters  $(m, p)$  of the system allows that  $|R_\chi| > 1$  then by rotating the external magnetic field one can tune the Josephson junction from the 0- to  $\pi$ -junction. Note that contrary to the  $\pi$ -junction based on d-wave high- $T_c$ -superconductors junctions based on MSC system can be varied from 0- to  $\pi$ -junction by simple changing orientation of external magnetic field. Even in the case  $|R_\chi| < 1$ , when only 0-junction can be realized, by tuning  $\theta$  one can make significant changes of the Josephson current.

The proposed MSC-I-MSC junction opens new physical possibilities if spatial and time variations of  $\theta(x, y, t)$  are realized. It is to expect that magnetic and electric fields, applied in the contact, will change both phases  $\theta(x, y, t)$  and  $\varphi(x, y, t)$ . The elaboration of these effects in a form of coupled equations for  $\varphi(x, y, t)$  (a modified Ferrell-Prange equation) and for  $\theta(x, y, t)$  is the matter of future researches.

In conclusion, we demonstrate that in magnetic superconductors with spiral magnetic order the Josephson current depends on new degrees of freedom: (1) the relative magnetic phase (orientation)  $\theta = \theta_L - \theta_R$  of the exchange fields (magnetizations) on the barrier, and (2) on the helicities of the left and right spiral, i.e. on  $\chi (= \chi_L \chi_R) = \pm 1$ . In some range of parameters  $\Delta$ ,  $h_{ex}$  and  $Qv_F$  the  $\pi$ -junction can be realized by tuning  $\theta$  in external magnetic field. Even in the case of the 0-junction the Josephson current can be varied significantly by tuning  $\theta$  thus giving rise to interesting physics. Time and spatial changes of the magnetic phase  $\theta$  and the Josephson phase  $\varphi$  can be of potential interest for small-scale applications, of course if the MSC-I-MSC junction is realizable.

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